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Some prediction problems of an important class of non-stationary processes, namely periodically correlated processes were studied.

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A STUDY OF NON-STATIONARY STOCHASTIC PROCESSES WITH THEIR APPLICATIONS

FINAL REPORT

January 25, 1995

U. S. ARMY RESEARCH OFFICE

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STATEMENT OF THE PROBLEM

Many important time series arising in engineering and applied science are not stationary. Hence, the investigation of non-stationary processes is essential. A variety of interesting non-stationary classes of stochastic processes including the periodically correlated processes and almost periodic processes have been already introduced and studied. Another class of non-stationary processes, which has been introduced and studied, extends the classes we mentioned above [1]. This new class seems to have very useful applications. It is clear that helicopter noise, being the combination of the noises generated by the main and rear rotor, is not always periodically correlated. However, this process turns out to be in our newly introduced class of non-stationary processes [1,2,3].

The study of most classes of non-stationary stochastic processes mentioned above is by no means complete. It was the purpose of this project to study these important non-stationary stochastic processes and to explore their other applications in science and engineering. We also proposed to address some theoretical problems regarding the new class.

Let's be more specific and start with some definitions

<u>Definition.</u> A stochastic process X_n, with correlation function $R(m,n)$ is called

a) Stationary if

$$R(m,n) = R(m+1,n+1), \text{ for all } m,n \text{ in } Z,$$

b) Periodically Correlated (PC) with period q if

$$R(m,n) = R(m+q,n+q), \text{ for all } m,n \text{ in } Z.$$

c) Correlation Autoregressive (CAR) if there exist finitely many scalars a_j, $j= -p,\ldots -1,0,1,\ldots p$; such that

$$(*) \qquad R(m,n) = \sum_j a_j R(m+j,n+j), \quad \text{for all } m,n \in Z;$$

Examples. Clearly every stationary process is CAR. In fact, we can let $p=1$, $a_{-1}=0$, $a_1=1$ to see (*) holds. Any PC process with period q also satisfies (*), this time we may take $p=q$, $a_q=1$ and the rest of a_i's being 0. So they are also CAR. As another example consider the stochastic process $Y_n = a^n X_n$, where X_n is a stationary process and a is any complex number. One can easily see that correlation function of Y_n satisfies

$$R(m,n) = \frac{1}{|a|^2+1} R(m+1,n+1) + \frac{|a|^2}{|a|^2+1} R(m-1,n-1),$$

which means Y_n is CAR. As a final example one can examine that

$Z_n=nX_n$ satisfies

$$R(m,n) = 3R(m+1,n+1) - 3R(m+2,n+2) + R(m+3,n+3),$$

and hence is Z_n is again a CAR process.

Note. The CAR process of example in last two example are stationary or PC. This for example because they are not bounded. In fact, because of the same reason CAR processes are in general not even harmonizable.

SUMMARY OF IMPORTANT RESULTS OBTAINED

We studied some prediction problems of an important class of non-stationary processes, namely periodically correlated processes.

In the paper #1 of the following list we develop a formula which gives the predictor explicitly and can be then used easily for the purpose of finding the predictor and its corresponding prediction error.

In the manuscript #2 of the following list which has appeared of a chapter in the book recent developments of prediction theory of Periodically Correlated processes is discussed for more applied audience.

In the paper #3 of the following list we show how our newly introduced class of Correlation Autoregressive Processes can be applied to model seismic and earthquake waves.

It is well-known that spectral domain of stationary stochastic processes is a complete function space, a fact which is crucial in the prediction theory of these processes. When it comes to non-stationary processes this problem is as important but unsolved. So the problem of weather the spectral domain of a harmonizable process is complete or not has been subject of study in several papers [4,5,6,7]. In the paper #4 of the following list we study this problem for the case of Periodically Correlated and Correlation Autoregressive Processes and obtain some characterization for their domain to be complete.

LIST OF PUBLICATIONS FROM THIS GRANT

1) A. G. Miamee; Explicit formulas for the best linear predictor and prediction error matrix of a periodically correlated process, SIAM J. Math. Anal. **24** (1993), 703-711

2) A. G. Miamee; On Recent Developments in Prediction Theory for Cyclostationary Processes" in Cyclostationarity in Communications and Signal Processing, (W.A. Gardner, Ed.) IEEE Press, New York,1994

3) G. R. Dargahi-Noubary and A. G. Miamee; Seismic waves and correlated autoregressive processes, J. Mathematical Geology **25** (1993), 671-688

4) A. G. Miamee and B. S. W. Schroeder; On completeness of the spectral domain of harmonizable processes Theory of Probability and Related fields (to appear)

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PARTICIPATING SCIENTIFIC PERSONNEL

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